

Numerical experiments with p F- and q D-strings: the formation of (p, q) bound states

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Abstract. We investigate the behaviour of (p, q) string networks, focusing on two aspects: (1) modelling more realistic (p, q) string networks than the Z_N networks used so far and (2) investigating the effect of long-range interactions on the evolution of the network. We model the network with no long-range interactions using two sets of fields, complex scalars coupled to gauge fields, with a potential chosen such that the two types of strings will form bound states. This way we can model junctions of 3 strings with different tension; in Z_N models used so far in simulations all the strings have identical tensions. In order to introduce long-range interactions we also study a network in which one of the scalars forms global strings.

We observe that in the absence of long-range interactions the formation of bound states has a significant influence on the evolution of the network. When long-range interactions are turned on the bound states are short-lived and have a minimal effect on the network evolution.

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1. Introduction

Recent observational data [1] strongly support the inflationary paradigm [2] as a successful solution to the shortcomings of the standard hot big bang model and the origin of density fluctuations, leading to the observed large-scale structure. Nevertheless, despite its success, inflation still remains a paradigm in search of a model. Successful inflationary models should thus be motivated by some fundamental physics. Having this in mind, inflation has been studied extensively in the framework of supersymmetric grand unified theories, where the end of an inflationary era is generically accompanied by cosmic string formation [3]. In addition, inflation faces some issues regarding its onset under generic initial conditions. The studies on the probability of the onset of inflation [4] indicate that it should take place in the deep quantum gravity regime. A

successful inflationary model could thus be obtained in the process of brane interactions, in the framework of brane cosmology within string theory [5].

String theory, a consistent theory of quantum gravity, which is assumed to be relevant during the early stages of our universe, leads to a number of insights into the physics of inflation. One crucial issue in cosmological models inspired by string theory is that compactification to four space-time dimensions leads to a number of scalar fields and moduli. On the one hand, moduli could play the rôle of the inflaton field, provided they do not roll quickly. On the other hand though, runaway moduli would destroy any consistent cosmological model. Thus, one requires moduli stabilisation, as achieved for instance within the KKLT scenario [6]. Among the various proposed brane inflation models [7], the KKLMNT model [8] is a brane-anti-brane annihilation scenario in the warped geometry. In this model, the inflaton field is associated with the relative position of branes in the compactified space, and all moduli are stabilised at the exit from inflation. Brane annihilation releases energy which can then reheat the universe, thus the radiation-dominated phase of the standard hot big bang cosmology can take place.

Consider a Type IIB string theory in 10 space-time dimensions. Brane annihilations allow the survival only of three-dimensional branes [9], one of which plays the rôle of our universe, with the copious production of fundamental (F-strings) and Dirichlet D1-branes (D-strings). Such strings are of cosmological size and they could play the rôle of cosmic strings [10]; they are referred to in the literature as cosmic superstrings [11]. Individually, the F- and D-strings are $\frac{1}{2}$ -BPS objects, which however break a different half of the supersymmetry each. In a number of successful brane-inflation models, as for example in the KKLMNT model, a spectrum of (p, q) strings, bound states of p F- and q D-strings, with reduced tensions was found. The bound (p, q) states have tension

$$\mu_{(p,q)} = \mu_F \sqrt{p^2 + q^2/g_s^2} , \quad (1)$$

where μ_F denotes the effective fundamental string tension after compactification and g_s stands for the string coupling. The (p, q) bound states are still $\frac{1}{2}$ -BPS objects. The presence of stable bound states implies the existence of junctions, where two different types of string meet at a point and form a bound state leading away from that point.

Traditionally, cosmic strings have been assumed to share the characteristics of type-II Nielsen-Olesen (NO) vortices [12] in the Abelian Higgs model. The main differences between these ordinary cosmic strings and cosmic superstrings are the following: (i) the intercommutation probability for ordinary strings is equal to 1, whereas it is smaller (often much smaller) than 1 in the case of superstrings [13]; (ii) ordinary string networks consist of (sub-horizon sized) loops and (super-horizon sized) long strings, whereas cosmic superstring networks have also junctions at which three string segments meet; (iii) all strings in an ordinary string network have the same tension, whereas there is a whole range of tensions for superstrings. The last two of these features are shared with type-I vortices in the Abelian Higgs model [14], but in contrast with them, cosmic superstrings have two integer-valued charges p and q .

These differences are expected to imply a different evolution of a cosmic superstring network as compared to that of ordinary strings. In particular, a question which has early been addressed is whether such a network will eventually reach scaling, or whether it will freeze [15], leading to predictions inconsistent with our observed universe. The reader can find in the literature a number of numerical experiments [16, 17, 18, 19, 20, 21, 22, 23] with cosmic superstrings, each of them at a different level of approximation, as well as analytical studies [24]. These distinct features are expected to lead to different observational and cosmological consequences.

The aim of our work is to build a simple field model of (p, q) bound states, in analogy with the Abelian Higgs model used to investigate the properties of ordinary cosmic string networks, and to study its properties using lattice simulations. We are mainly interested in the overall characteristics of the network, and therefore we focus on the total energy of the network, rather than the dynamics of individual strings. In Section 2, we briefly describe our model. In Section 3, we discuss the (p, q) string spectrum. In Section 4, we describe our numerical approach to studying the (p, q) string spectrum. In Section 5, we present our results from numerical experiments with small and large simulations. We round up with our conclusions in Section 6.

2. The model

We want to build a model of (p, q) strings which captures the main features of the string theory model and is amenable to study via lattice simulations. Therefore, we need a system which features:

- Two different species of cosmic strings. We realise that by including two sets of fields of the Abelian Higgs model.
- The formation of bound states. We realise that by introducing a coupling of the scalar fields via the potential.
- One non-BPS species of cosmic string. Such strings have long-range interactions (regardless of their orientation) and this can be realised by having the second type of string be the topological defect of a scalar field with a global $U(1)$ symmetry.

In the case where both species of cosmic strings are BPS, the action reads:

$$\mathcal{S} = \int d^3x dt \left[-\frac{1}{4}F^2 - \frac{1}{2}(D_\mu\phi)(D^\mu\phi)^* - \frac{\lambda_1}{4}(\phi\phi^* - \eta_1^2)^2 - \frac{1}{4}H^2 - \frac{1}{2}(D_\mu\phi)(D^\mu\phi)^* - \frac{\lambda_2}{4}\phi\phi^*(\chi\chi^* - \eta_2^2)^2 \right], \quad (2)$$

where ϕ and χ are two complex scalar fields, and we have used a compact notation for the covariant derivative D_μ , so that

$$\begin{aligned} D_\mu\phi &= \partial_\mu\phi - ie_1A_\mu\phi, \\ D_\mu\chi &= \partial_\mu\chi - ie_2C_\mu\chi. \end{aligned} \quad (3)$$

In order to avoid confusion we will refer to the ϕ field as the “Higgs” and to the χ field as the “axion”, even though both fields are Higgs-like. The scalars are coupled

to two different U(1) gauge fields A_μ and C_μ , with coupling constants e_1 and e_2 and field strength tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$, respectively. The scalar potentials are parameterised by the positive constants $\lambda_1, \lambda_2, \eta_1, \eta_2$. In the case of a non-BPS species of string, we remove the second gauge field by setting $e_2 = 0$.

The classical equations of motion for the fields follow from the action (2),

$$\begin{aligned}\partial_\mu F^{\mu\nu} &= 2e_1 \text{Im} \phi^* D^\nu \phi, \\ \partial_\mu H^{\mu\nu} &= 2e_2 \text{Im} \chi^* D^\nu \chi, \\ D_\mu D^\mu \phi &= -2\lambda_1 (\phi^* \phi - \eta_1^2) \phi \\ D_\mu D^\mu \chi &= -2\lambda_2 \phi^* \phi (\chi^* \chi - \eta_2^2) \chi.\end{aligned}\tag{4}$$

Let us look at the potential terms for the two scalar fields. The overall value of the potential for the χ field depends on the value of the ϕ field at that location. If ϕ is in the vacuum, $\phi\phi^* = \eta_1^2$, the potential has the usual Mexican hat form, while at the core of a ϕ vortex we have $\phi\phi^* = 0$. If a χ vortex passes through the core of a ϕ vortex, the potential energy of the χ field at the core of the vortex is (almost) eliminated, so the energy of the vortex is reduced. It is therefore favourable for the two vortices to form a bound state.

It is easier to illustrate how this works if we look at an extreme situation, where the thickness of the axion string is much smaller than that of the Higgs string. Let us consider a Higgs string located at the origin of the $x - y$ plane and oriented along the z direction. We can use the radial profile of the Higgs field, $\phi(r)$, to determine the radial profile of the function, $\phi\phi^*$, which multiplies the axion field potential. Since the axion field potential energy is non-zero only in a very small region, the function $|\phi(r)|^2$ will be the potential seen by the axion string. Therefore, the axion string will prefer to move to a location where $|\phi(r)|^2$ is minimised, and that place is the core of the Higgs string. This is shown schematically in Fig. 1.

In an actual simulation we observe that when two strings intersect, they remain “snagged” and then the segments coming into the junction align themselves to allow the bound state segment to grow in length. The two processes working against this tendency are the long-range interactions between the strings and the motions of the unbound parts of the strings. Snapshots of such bound string states, obtained in our numerical experiments, are shown in Fig. 2.

3. The (p, q) string spectrum

The spectrum of the (p, q) strings was studied for various set-ups, some of which we would like to mention here. First of all, the most commonly used formula for (p, q) string tension, Eq.(1), represents the BPS bound for an object carrying the charges of p F -strings and q D -strings, the more general result being that the mass per unit length of such an object satisfies the inequality (see, Ref. [25]):

$$\frac{M}{L} \geq \frac{\sqrt{p^2 + q^2/g_s^2}}{2\pi\alpha'}.\tag{5}$$

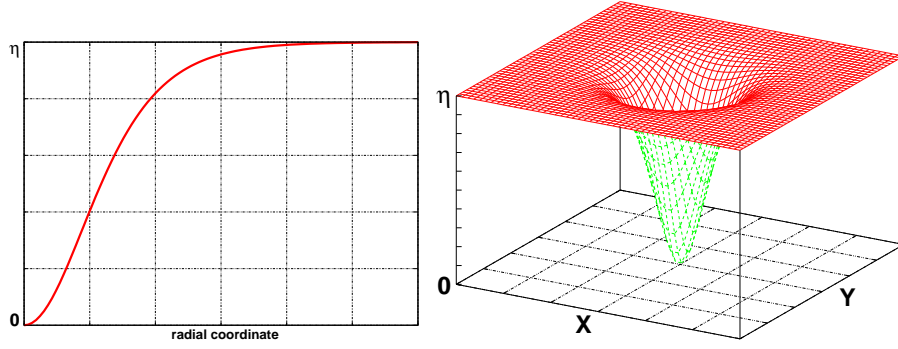


Figure 1. Left panel: Radial field profile for the Higgs field. Right panel: Effective potential as seen by a very thin axion string. We can use the radial field profile of the Higgs field to determine a spatial potential for the axion string. The location corresponding to minimum energy for the axion string is the core of the Higgs string.

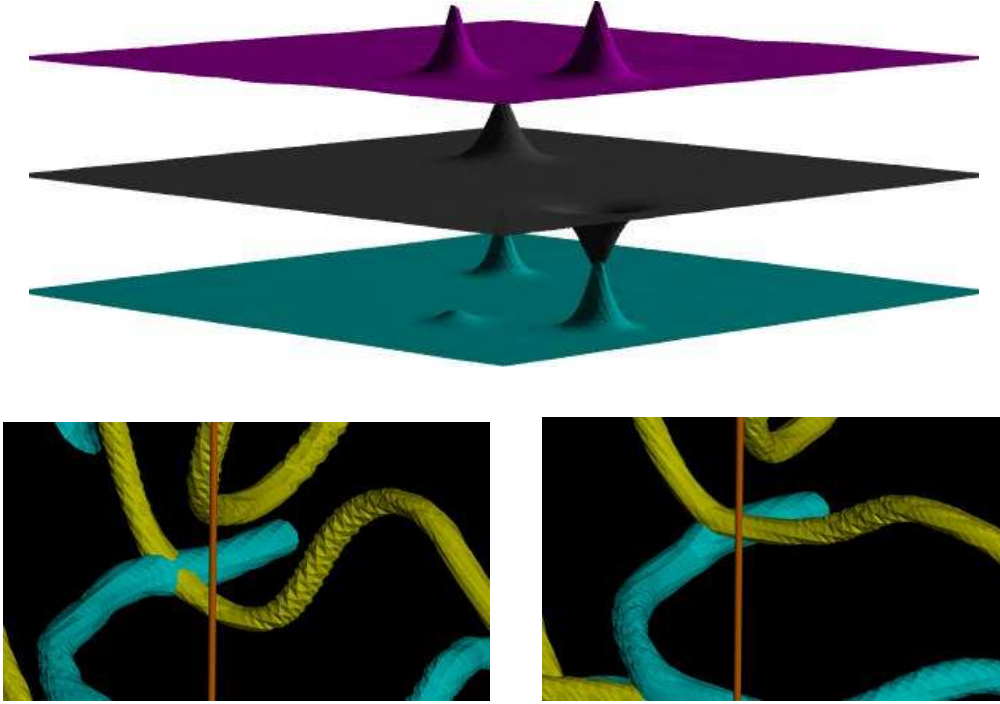


Figure 2. Top: Bound state of vortices in two dimensions. From top to bottom, the layers show the axion field $|\chi|$, the magnetic flux $\vec{\nabla} \times \vec{A}$ that couples to the Higgs, and the Higgs field $|\phi|$. The location of the defect on the right coincides in the Higgs and the axion fields, suggesting that they form a bound state. The binding is evident from the way the axion vortex drags the Higgs vortex with it, as it is pulled towards the anti-vortex on the left. The local Higgs vortex on the background does not move as its interactions with the other vortices are exponentially suppressed. Bottom: In three dimensions, bound states split because of long-range interactions and the motion of the strings. As a result, bound states constitute only a small fraction of the entire string network. The two pictures show a bound state in a process of splitting.

If we consider a more realistic model with explicit stabilisation of the compactification, the strings formed at the end of brane inflation will be localised at the bottom of a warped throat [26], since the tension of the cosmic string is given by the value of the warp factor [8]:

$$\mu_1 = \frac{1}{2\pi g_s \alpha'} \left(\frac{r_0}{R} \right)^2 . \quad (6)$$

It was noted in Ref. [27] that the D1 branes placed at the bottom of a warped throat embedded in a flux compactification cannot be BPS. For such an object with q units of D1 charge the expression of the tension was found to be [28]:

$$\mu_{1,q} \sim \sin \left(\frac{\pi q}{M} \right) , \quad (7)$$

where M is the number of units of R-R flux through the \mathbf{S}^3 at the bottom of a K-S throat, [29].

A different class of objects with p F -strings and q D -strings units of charge was studied in [30]. Such an object is a $D3$ -brane wrapped on the collapsing \mathbf{S}^2 of a K-S throat, and the spectrum of tensions for such an object is:

$$\mu_{(p,q)} = \frac{1}{2\pi \alpha'} \sqrt{\left(\frac{bM}{\pi} \right)^2 \sin^2 \left(\frac{\pi (p - qC_0)}{M} \right)^2 + \frac{q^2}{g_s^2}} . \quad (8)$$

Thus, the most commonly used formula for the spectrum of the (p, q) string tension, Eq.(1), is exact only in the BPS limit, while for more general situations we expect departures from this theoretical prediction. In order to make a comparison of our model with the theoretical formula, Eq. (1), we will numerically determine the tension of the (p, q) strings for a range of values of the charges and then fit the numerical data with the expression given in Eq. (1).

The systems we use to model the (p, q) strings are different in a number of aspects from the ones used so far, and we believe that our systems bring a number of improvements and allow for additional effects to be studied.

The systems used so far to simulate (p, q) string networks allow for the formation of 3-string [22] (or more generally N-string [19]) junctions, in which all strings that meet at a point have equal tension. In our models (constructed along the lines of [20]) the bound states have different tension than the single-charge strings, following the original string theory model more closely. Our models also allow to individually set the long-range interaction of each species of cosmic strings. As Eq. (8) suggests, for cosmic strings at the bottom of a K-S throat the F-string is not BPS while the D-string is. We therefore expect the different components of the (p, q) string to exhibit different types of long-range interactions. Also, as pointed out in Ref. [26], in models where the standard model branes are located at orbifold fixed points, not all bulk fields that couple to the cosmic strings of the model survive the orbifold projection. There will also be other fields present, coming from higher-rank form-fields integrated over collapsed cycles, which will mediate the interactions of cosmic strings obtained from higher-dimensional branes wrapping the same collapsed cycles. We therefore want to

allow for the possibility to independently choose the long-range interactions between the different species of cosmic strings.

4. Numerical analysis of the (p, q) string spectrum

In this section we construct the spectrum of (p, q) strings formed in our model and compare it with the one predicted by string theory, Eq.(1). We will start by considering an isolated defect and make an ansatz for the radial field profiles in analogy with the NO vortex [12]. In fact, the model consists of two NO vortices coupled via the scalar field potential.

The cosmic string we consider is an infinitely long, straight cosmic string, so we will choose cylindrical coordinates with the z axis oriented along the string. The field configuration is independent of the coordinate along the string, z , so we are effectively describing a 2-dimensional vortex.

The static, cylindrically symmetric ansatz for the fields of a string with charges (p, q) is that of two NO vortices with winding numbers p and q :

$$\begin{aligned}\phi(r, \varphi) &= f(r) e^{-ip\varphi} \\ \vec{A}(r) &= \frac{a(r)}{r} \hat{e}_\varphi \\ \chi(r, \varphi) &= h(r) e^{-iq\varphi} \\ \vec{C}(r) &= \frac{b(r)}{r} \hat{e}_\varphi ,\end{aligned}\tag{9}$$

where \vec{A} and \vec{C} denote the spatial components of the gauge fields. The corresponding equations of motion are:

$$\begin{aligned}\frac{\partial^2 a}{\partial r^2} - \frac{1}{r} \frac{\partial a}{\partial r} - e_1 (p - e_1 a) f^2 &= 0 \\ \frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} - \frac{(p - e_1 a)^2}{r^2} - \lambda_1 (f^2 - \eta_1^2) f - \frac{\lambda_2}{2} (h^2 - \eta_2^2)^2 f &= 0 \\ \frac{\partial^2 b}{\partial r^2} - \frac{1}{r} \frac{\partial b}{\partial r} - e_2 (q - e_2 b) h^2 &= 0 \\ \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} - \frac{(q - e_2 b)^2}{r^2} - \lambda_2 (h^2 - \eta_2^2) f^2 h &= 0 .\end{aligned}\tag{10}$$

From the above equations, we see that at $r = 0$ all four functions $a(r)$, $b(r)$, $f(r)$ and $h(r)$ vanish, while at $r = \infty$ the functions go asymptotically to:

$$a(r) \rightarrow \frac{p}{e_1} , \quad f(r) \rightarrow \eta_1 , \quad b(r) \rightarrow \frac{q}{e_2} , \quad h(r) \rightarrow \eta_2 .\tag{11}$$

We are interested in obtaining the energy per unit length of the bound states, so it will be more convenient to calculate the expression of the Hamiltonian in terms of the 4 functions given above, and perform a gradient flow to obtain the minimum energy configuration. The radial profiles of the fields will appear as a by-product of the energy minimisation process.

The Hamiltonian density has the expression:

$$\begin{aligned} \mathcal{H} = & \frac{1}{2} \left| \frac{\partial f}{\partial r} \right|^2 + \frac{1}{2} (p - e_1 a)^2 \frac{f^2}{r^2} + \frac{1}{2} \left| \frac{1}{r} \frac{\partial a}{\partial r} \right|^2 + \frac{\lambda_1}{4} (f^2 - \eta_1^2)^2 \\ & + \frac{1}{2} \left| \frac{\partial h}{\partial r} \right|^2 + \frac{1}{2} (q - e_2 b)^2 \frac{h^2}{r^2} + \frac{1}{2} \left| \frac{1}{r} \frac{\partial b}{\partial r} \right|^2 + \frac{\lambda_1}{4} f^2 (h^2 - \eta_2^2)^2 . \end{aligned} \quad (12)$$

In order to perform the gradient flow we start by discretising the radial direction into N intervals. The 4 functions become 4 sets of $N + 1$ variables whose values will need to be determined, so that the expression of the Hamiltonian integrated along the radial direction can be minimised. In the discrete version the integral is replaced by a sum, namely

$$\begin{aligned} H = 2\pi \int_0^\infty r dr \mathcal{H}(r) \rightarrow \\ 2\pi \sum_{k=0}^{N-1} \left\{ \left(k + \frac{1}{2} \right) \frac{[f_{k+1} - f_k]^2}{2\Delta x} + \frac{[a_{k+1} - a_k]^2}{(2k+1)\Delta x^3} \right. \\ + \left(k + \frac{1}{2} \right) \frac{[h_{k+1} - h_k]^2}{2\Delta x} + \frac{[b_{k+1} - b_k]^2}{(2k+1)\Delta x^3} \\ + 2\pi \sum_{k=1}^{N-1} \left\{ \frac{[p - e_1 a_k]^2}{2k\Delta x} f_k^2 + \frac{[q - e_2 b_k]^2}{2k\Delta x} h_k^2 \right. \\ \left. \left. + \frac{\lambda_1 k \Delta x}{4} [f_k^2 - \eta_1^2]^2 + \frac{\lambda_2 k \Delta x}{4} f_k^2 [h_k^2 - \eta_2^2]^2 \right\} \right\} . \end{aligned} \quad (13)$$

In a numerical calculation the radial direction cannot be infinite, so we will have to choose it to be large as compared to the size of the defect core. The values of the functions at the ends of the intervals $k = 0$ and $k = N$ will not evolve, they will be kept fixed at their core and asymptotic values, respectively. To find the equations of motion for the rest of the $4N - 4$ variables we calculate the variation of the Hamiltonian with respect to each variable for $p = 1 \dots N - 1$. One can then find the minimal energy configuration by evolving the equations

$$\frac{\partial f_p}{\partial \tau} = -\frac{\delta H}{\delta f_p} \quad \dots \quad \frac{\partial b_p}{\partial \tau} = -\frac{\delta H}{\delta b_p} , \quad (14)$$

starting with an appropriate initial configuration and keeping the values of the functions at the end of the interval fixed. The initial configuration we use is a linear radial profile starting with zero at $r = 0$ and ending with the corresponding asymptotic value at $r = N\Delta x$. The results, together with a fit of the form 1 are shown in Fig. 3.

5. Results from numerical experiments

To investigate the formation and evolution of bound states in a string network, we carried out simulations of the classical time evolution on lattices with two and three spatial dimensions. We carried out these simulations in pairs, using the same initial conditions and parameters of the Lagrangian in both cases. In the first case, the Higgs

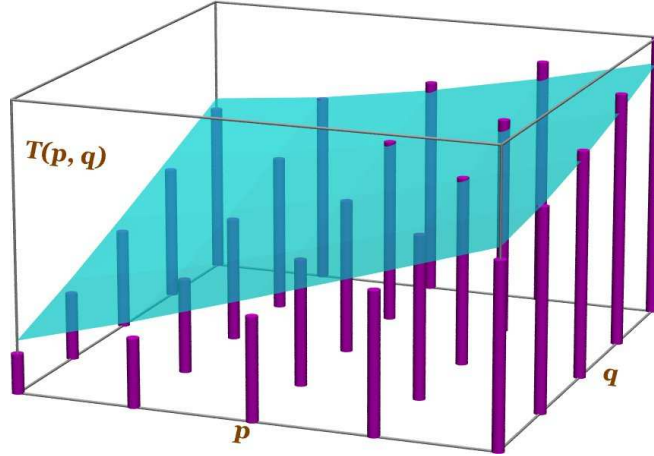


Figure 3. Fit of the square-root spectrum. The vertical bars are the string tension values calculated via the energy minimisation method. The surface is the function $T(p, q) = \sqrt{ap^2 + bq^2}$ computed with the parameters a and b determined from the fit.

and the axion have both local U(1) symmetry. In the second case, the axion field has a global U(1) symmetry, and therefore the axion strings display long-range interactions.

The details of the lattice discretisation of these equation are presented in Appendix B. We chose all parameters of the Lagrangian to have “natural” values,

$$\eta_1 = 1, \lambda_1 = 1, e_1 = 1, \eta_2 = 1, \lambda_2 = 1, e_2 = 1. \quad (15)$$

The lattice spacing was $\delta x = 0.45$, roughly half the characteristic length scale set by the masses of the scalar and gauge field in the broken phase. The time step was $\delta t = 10^{-2}$. The simulations were carried out with three different box sizes 200^3 , 256^3 and 400^3 .

We used initial conditions in which the gauge potentials A_μ and C_μ are zero everywhere, so we have vanishing electric and magnetic fields. We also set the initial values of the time derivatives of the scalar fields to zero. The values of the fields themselves were set at the maximum of the potential $\phi = 0$, $\chi = 0$. To this configuration we added, depending on the problem studied, a small-amplitude random white-noise component, or a small-amplitude component with phases chosen such that the network of cosmic strings will form in a particular configuration. For such an initial configuration the Gauss constraint (B.4) is automatically satisfied.

The white-noise initial conditions correspond to the symmetric phase with zero correlation length. As the system starts to evolve, the U(1) symmetries get spontaneously broken, and strings are formed by the Kibble mechanism [31]. Because the gauge fields were set to zero, no flux trapping [32] takes place, in contrast with thermal initial conditions. The initial white-noise configuration excites all modes, with half-wavelengths up to the lattice spacing, and in order to see the formation and evolution of a network of defects we have to drain the excess energy from the system. We achieved that by including a constant damping $\sigma = 0.2$ term for the scalar fields and an ohmic term for the gauge fields, such that both gauge fields satisfy the Gauss constraint (B.4)

during the entire simulation.

What we compare in the two networks is the fraction of the volume of the simulation box occupied by bound states. For completeness, we also track the fraction of the simulation box volume occupied by single charged strings. Our set-up allows us to start with initial conditions which will lead to the formation of a particular network configuration. Starting with a white-noise configuration, the total volume occupied by bound states is too small to lead to a sizable difference in their volumes, for the two types of string networks. This is also due to the particular form of the potential for the axion field. This potential vanishes at the core of a Higgs string and gets its largest value where the Higgs field is in the vacuum. Therefore, starting with white-noise initial conditions the axion strings prefer to form where the Higgs field is already in the vacuum. Thus, the formation of bound states is not favoured. Even when we use a very large simulation box, 400^3 , we encountered situations during the network evolution where no bound states were present. It is therefore essential to be able to choose the initial phases of the scalar fields such that the phases have various degrees of “alignment”, so that the formation of bound states is favoured. In this sense, white-noise corresponds to no alignment at all, while choosing identical initial conditions for both the Higgs and the axion fields results in a network with all strings formed in bound states.

Our aim was to observe the evolution of the network of (p, q) strings, paying special attention to the evolution of bound states. We observed that in the case of local-global networks the bound states do not have a significant influence on the evolution of the network. The long-range interaction of the global strings prevents bound states of significant size from being long lived. This is not the case for local-local string networks in which we observed long lived bound state string segments. These bound states have a significant effect on the evolution of the network.

More quantitatively, Fig. 4 shows the fraction of the simulation box occupied by the Higgs strings, the axion strings, and their bound states. If we choose white-noise initial conditions the bound states formed are longer-lived in the local-local network than in the local-global one, but the small overall volume of the bound states did not allow us to obtain results of any statistical significance. Nevertheless, we observed the same trend at all lattice sizes: The volume occupied by the bound states is larger in the case of a local-local network and the total energy is larger for the local-global network. For the largest box we also encountered a situation in which the network did not feature any bound states. While this was certainly a finite-size effect, in the local-local network evolving from identical initial conditions the bound states survive past the point where the local-global bound states disappear.

However, if we choose the Higgs and the axion fields to be partially aligned in the initial configuration the volume of the bound states formed is large enough to obtain a statistically significant difference. We obtain the partial alignment by choosing the initial configurations identical and then rotating the one for the axion field by 90° . The fields are identical along the rotation axis. We performed a number of simulations starting from various initial conditions and the results are summarised in Fig. 4. We

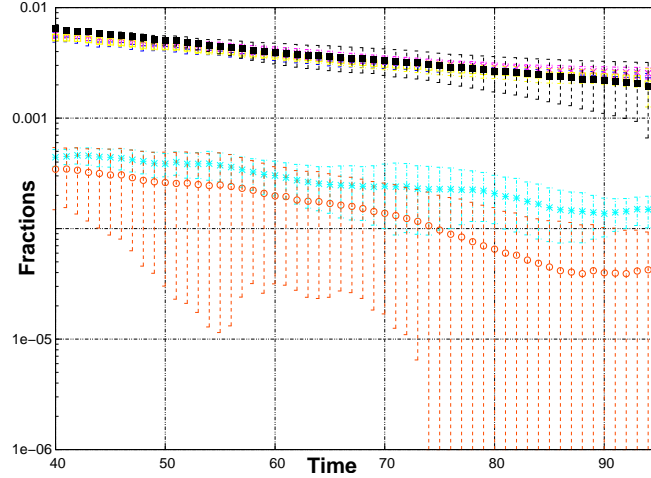


Figure 4. (color online) The volume of the bound states for local-local (blue) and local-global (orange) networks. The bound states of the local-global network show a much larger spread in volume as a consequence of their short lifetime. The curves at the top represent the volumes of the simulation box occupied by each species of cosmic string. In this example all simulations were performed in a 200^3 box. The volumes occupied by each type of string and the bound states are averaged over 10 different initial conditions.

observe that the local-local network forms bound states that are much longer-lived than those of the local-global network. The short lifetime of the bound states of the local-global network results in a much larger spread of the bound state volume, and it is common to encounter situations where no bound states are present.

The most convincing evidence comes from analysing the reverse problem, namely that of a bound state splitting as a result of the long-range interactions between strings. We performed a number of simulations in which we started with an already-formed network with all the strings carrying $(1,1)$ charge. We achieved that by choosing identical (“fully aligned”) initial configurations for the Higgs and the axion fields. These results, presented in Fig. 5, show most clearly that the effect of the long-range interactions is to cause the bound states to split. In the absence of long-range interactions the strings remain in the $(1,1)$ state throughout their entire evolution. The total physical volume occupied by the bound states is identical with the volume of the thinner strings. Due to our particular choice of binding potential the strings do not have identical thickness. There are only minor deviations in the initial phase of the evolution when the strings are not yet fully formed.

The main result of the simulations is that the long-range interactions between global strings make the bound states of the local-global network short-lived. The bound states turn out not to have any significant contribution to the evolution of the network; the evolution is dominated by the long-range interactions between strings. In the case of a local-local network, the absence of long-range interactions allows for the formation of bound states of larger volume and these bound states do have an important influence

on the evolution of the network.

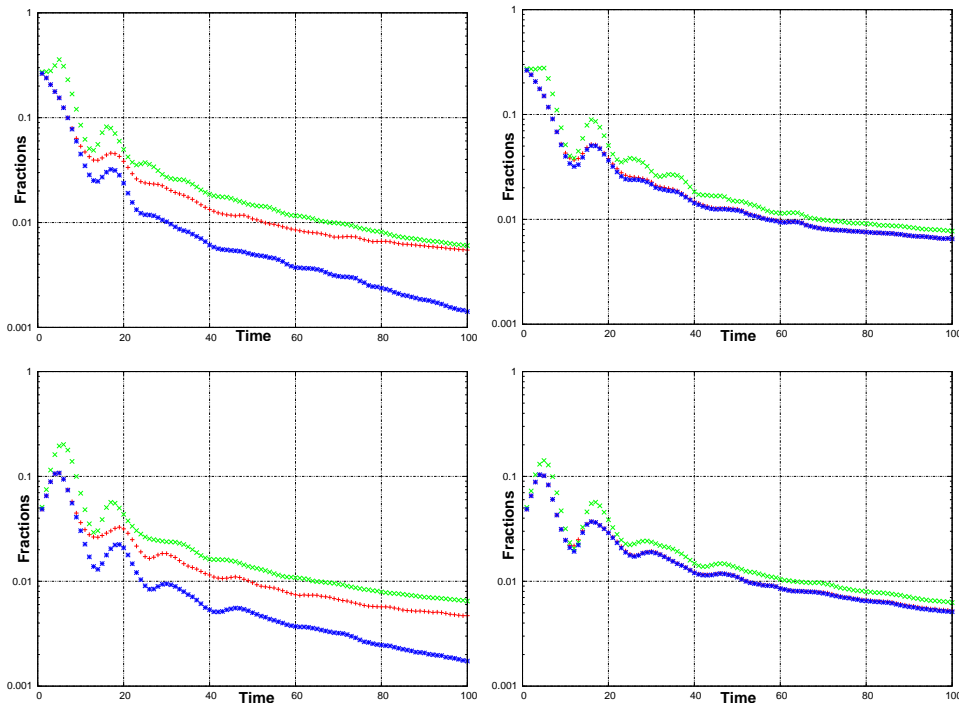


Figure 5. (color online) The total physical volume of the simulation box occupied by Higgs strings (green), the axion strings (red), and their bound states (blue). On the left we show two examples of volume evolutions for local-global networks and on the right the corresponding values for a local-local network starting from the same initial conditions as the local-global ones. We see that in the local-global case a significant fraction of the strings become unbound early in the simulation. In the local-local case the volume of the bound states is identical with the one of the thinner (axion) strings.

Another way to visualise the competing effects of the bound states and the long-range interactions is presented in Fig. 6. In this example we set the initial conditions such that there was only one pair of local and one pair of global strings. The strings form in pairs of opposite orientation because of the periodic boundary conditions. We arranged the configuration such that the attractive interactions between global strings resulted in their motion towards the local ones, with the aim to find out whether the formation of the bound states can stop the motion of the global strings.

We performed the simulations in boxes of various sizes and with different winding numbers for the local strings. In each case we observed that the long-range interactions caused the bound states to split. The global strings moved towards the local ones and crossed them, forming bound states in this process. These bound states then split as the global strings continued to move towards each other. Finally they collided and annihilated.

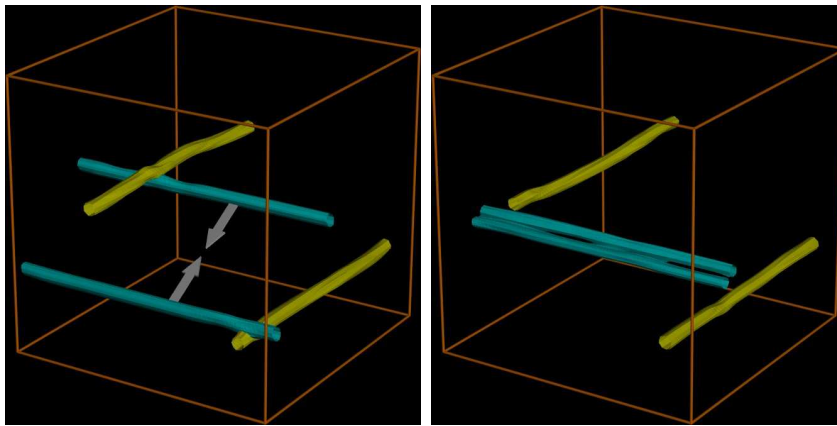


Figure 6. The bound states do not survive the long-range interaction of the global strings.

6. Conclusions

In this paper we have analysed the evolution of (p, q) string networks in an attempt to observe the influence that the bound states have on the evolution of the network. The (p, q) string network was modelled using two sets of Abelian Higgs fields. We studied two models, one in which both species of string have only short-range interactions and another one in which one species of string features long-range interactions. Our aim was to observe the competition between the binding and the long-range interactions in an attempt to see which effect will have a bigger influence on the evolution of the string network.

We started out with a simple set-up of local and global vortices in two space dimensions, where we could see that the two types of vortices form bound states and the long-range interactions between the global strings will drag the local vortices along with the global ones. The local vortices did not move as they displayed only short-range interactions.

We then continued with a three-dimensional model in which we studied the evolution of cosmic superstrings starting from two types of initial conditions. In one case we looked at only two pairs of cosmic superstrings, one local and one global, oriented at right angles. The initial positions were chosen such that the attractive interaction of the global strings would cause them to intersect the local ones. This way we could see whether the long range interaction will overcome the formation of bound states of the two string species. We repeated the simulation choosing local strings with different windings around the periodic simulation box, such that the local and the global strings would intersect in more than one point. In both cases we observed that the long-range interaction wins in the sense that the bound states split and the global strings move towards each other and annihilate.

The other case we studied was the formation and evolution of a network starting with various initial conditions. The evolution of the string networks suggests that the

long-range interactions have a much more important rôle in the network evolution than the formation of bound states. In the local-global networks the bound states tend to split as a result of the long-range interactions, resulting in two networks that evolve almost independently. The formation of short-lived bound states and their subsequent splitting only increases the small-scale “wiggleness” of the local strings. In the case of a local-local network, the absence of long-range interactions allows the bound states to be much longer-lived and significantly influences the evolution of the string network.

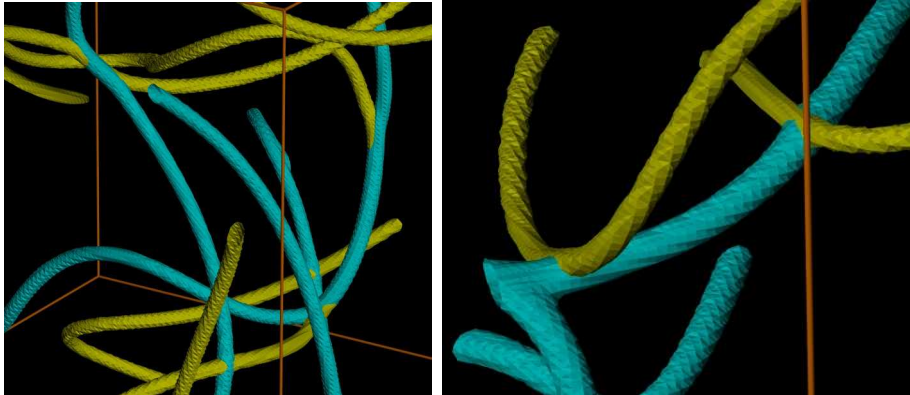


Figure 7. Left: Bound states for local-local (p, q) strings. Right: Bound states for local-global (p, q) strings.

In an attempt to express these observations in a more quantitative manner we plotted the fraction of the spatial volume occupied by the bound states for each type of a network, as well as the total energy of the string network as a function of time. The observations we mentioned above translate into a larger volume occupied by the bound states, in the absence of long-range interactions. The energy of the local-global network is larger than that of the local-local one, since the energy of the global strings is not localised and therefore the formation of bound states has a smaller effect on the total energy of the network.

Our results suggest that if in a (p, q) string network one species of cosmic string features long-range interactions, the bound states will not survive and we have two networks evolving (almost) independently. We did not address the question of scaling in this work, but since the bound states split due to the long-range interactions, we expect such a network to scale rather than freeze.

Our study included only two extreme cases, that of local-local and of local-global networks. While we believe that the models we use represent an improvement over the previous types of models considered, as far as the tension of the strings and their bound states is concerned, there is still room for improvement in modelling the long-range interactions.

As it was shown in Ref. [33], the long-range interactions between the cosmic strings can be much more accurately modelled by including a rank-2 antisymmetric tensor field that couples to the magnetic flux trapped at the core of the string via a Chern-Simons

term. It was also shown that, depending on the strength of the Chern-Simons coupling, the interaction mediated by the rank-2 tensor field interpolates between that of local and global defects. When the coupling is very small the interaction is negligible, while at strong coupling the magnetic flux at the core of the string is almost eliminated and the network behaves like a global string network. We therefore expect the interplay between the effects of binding and those of long-range interactions to be more subtle as one changes the strength of the Chern-Simons coupling. At small coupling we expect the evolution of the network to be dominated by the formation of bound states, while at large coupling the effect of bound states is negligible, and one gets two species of strings evolving almost independently.

We believe that the models presented in this paper represent a step forward in understanding the behaviour of a cosmic superstring network formed at the end of brane inflation in flux compactification scenarios. In a future work, we plan to use the models presented above to address the issue of scaling in a (p, q) string network.

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Appendix A. Equations for energy minimization

Varying the Hamiltonian Eq.(13) with respect to the values of the fields at each point along the discretized radial direction one obtains:

$$\begin{aligned} \frac{\delta H}{\delta f_p} = & \left(p - \frac{1}{2}\right) \frac{f_p - f_{p-1}}{\Delta x} + \left(p + \frac{1}{2}\right) \frac{f_p - f_{p+1}}{\Delta x} \\ & + \frac{[n - e_1 a_p]^2}{p \Delta x} f_p + p \Delta x \lambda_1 [f_p^2 - \eta_1^2] f_p + \frac{p \Delta x \lambda_2}{2} [h_p^2 - \eta_2^2]^2 f_p, \end{aligned} \quad (\text{A.1})$$

$$\frac{\delta H}{\delta a_p} = -\frac{e_1 [n - e_1 a_p]}{p \Delta x} f_p^2 + \frac{1}{p \Delta x^3} \left[\frac{a_p - a_{p-1}}{p - \frac{1}{2}} + \frac{a_p - a_{p+1}}{p + \frac{1}{2}} \right], \quad (\text{A.2})$$

$$\begin{aligned} \frac{\delta H}{\delta h_p} = & \left(p - \frac{1}{2}\right) \frac{h_p - h_{p-1}}{\Delta x} + \left(p + \frac{1}{2}\right) \frac{h_p - h_{p+1}}{\Delta x} \\ & + \frac{[m - e_2 b_p]^2}{p \Delta x} h_p + p \Delta x \lambda_2 [h_p^2 - \eta_2^2] f_p^2 h_p, \end{aligned} \quad (\text{A.3})$$

$$\frac{\delta H}{\delta b_p} = -\frac{e_2 [m - e_2 b_p]}{p \Delta x} h_p^2 + \frac{1}{p \Delta x^3} \left[\frac{b_p - b_{p-1}}{p - \frac{1}{2}} + \frac{b_p - b_{p+1}}{p + \frac{1}{2}} \right]. \quad (\text{A.4})$$

One can then impose the appropriate boundary conditions depending on the winding number of phase of each scalar field. The values of the fields at each point along

the radial direction evolve according to Eq.(14) and the final configuration determines the field profiles (see Fig.(A1)) and the corresponding energy of the topological defect.

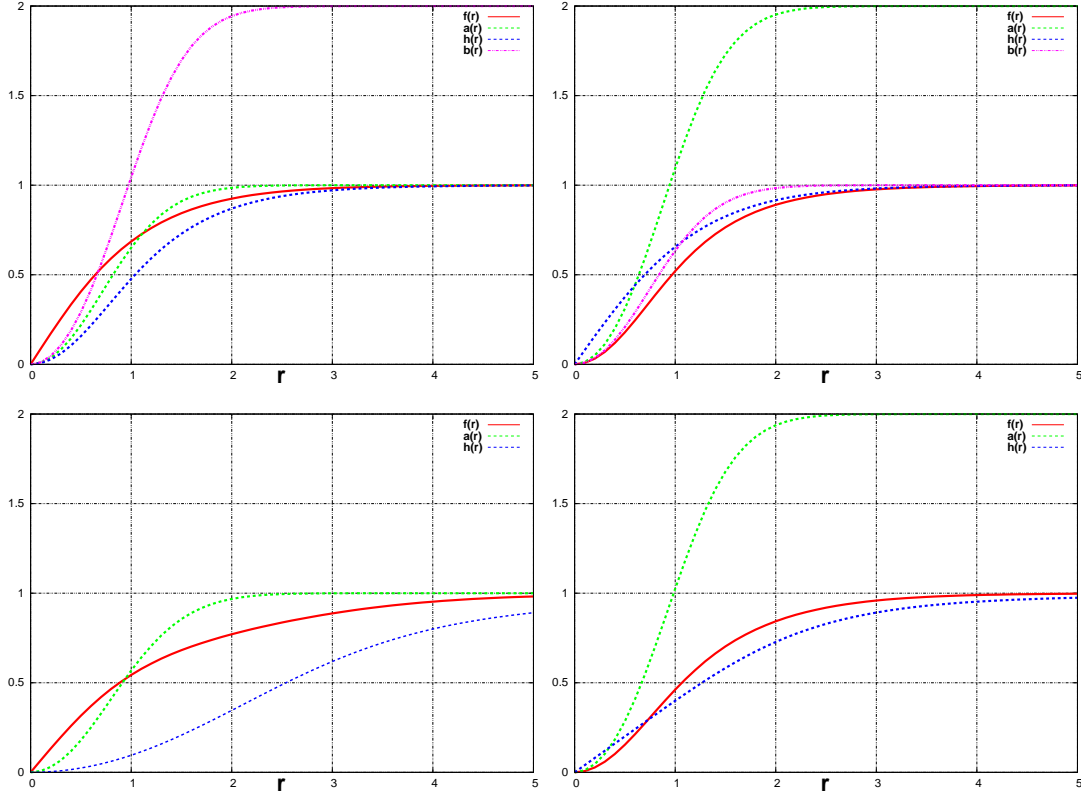


Figure A1. Field profiles for different winding numbers. The two upper panels show the field profiles for local-local vortices with winding numbers (1, 2) on the left and (2, 1) on the right. The lower panels show the field profiles for local-global vortices with winding numbers (1, 2) on the left and (2, 1) on the right. We denote by $f(r)$ the radial profile of the Higgs, by $a(r)$ the radial profile of the gauge field that couples to the Higgs, $h(r)$ for the axion and finally $b(r)$ for the gauge field that couples to the axion. For the local-global defect we simply set $b(r) = 0$ along the entire radial direction.

Appendix B. Lattice discretisation

In order to carry out numerical simulations described in Section 5, we discretised the equations of motion (4) in the standard leap-frog fashion. The scalar fields ϕ and χ were defined at the lattice sites, and their time derivatives $\dot{\phi}$ and $\dot{\chi}$ at temporal links between time slices. We used temporal gauge $A_0 = C_0 = 0$, and the spatial gauge fields \vec{A} and \vec{C} were represented by real numbers defined at links between lattice sites.

In order to write the equations of motion in a compact form, we define the link variables $U_i = \exp(-ie_1\delta x A_i)$ and $V_i = \exp(-ie_2\delta x C_i)$, and the forward and backward lattice derivatives of any function $f(\vec{x})$,

$$\Delta_i^\pm f(\vec{x}) = \pm \delta x^{-1} (f(\vec{x} \pm \vec{i}) - f(\vec{x})). \quad (\text{B.1})$$

We also define the lattice version of the covariant derivative

$$\begin{aligned} D_i^+ \phi(\vec{x}) &= \delta x^{-1} (U_{i,(\vec{x})} \phi(\vec{x}+\vec{i}) - \phi(\vec{x})), \\ D_i^- \phi(\vec{x}) &= \delta x^{-1} (\phi(\vec{x}) - U_{i,(\vec{x}-\vec{i})}^* \phi(\vec{x}-\vec{i})), \end{aligned} \quad (\text{B.2})$$

with U replaced by V if the derivative is acting on χ .

In this notation, the discretized equations of motion with the damping term σ are

$$\begin{aligned} \Delta_t \dot{A}_{i,(t,\vec{x})} &= -\sigma \dot{A}_{i,(t-\delta t,\vec{x})} - \epsilon_{ijk} \epsilon_{klm} \Delta_j^- \Delta_l^+ A_{i,(t,\vec{x})} - 2e_1 \text{Im} \phi_{(t,\vec{x})}^* D_i^+ \phi_{(t,\vec{x})}, \\ \Delta_t \dot{C}_{i,(t,\vec{x})} &= -\sigma \dot{C}_{i,(t-\delta t,\vec{x})} - \epsilon_{ijk} \epsilon_{klm} \Delta_j^- \Delta_l^+ C_{i,(t,\vec{x})} - 2e_2 \text{Im} \chi_{(t,\vec{x})}^* D_i^+ \chi_{(t,\vec{x})}, \\ \Delta_t \dot{\phi}_{(t,\vec{x})} &= -\sigma \dot{\phi}_{(t-\delta t,\vec{x})} + D_i^- D_i^+ \phi_{(t,\vec{x})} - \lambda_1 (|\phi_{(t,\vec{x})}|^2 - \eta_1^2) \phi_{(t,\vec{x})} \\ &\quad - \frac{\lambda_2}{2} (|\chi_{(t,\vec{x})}|^2 - \eta_2^2)^2 \phi_{(t,\vec{x})}, \\ \Delta_t \dot{\chi}_{(t,\vec{x})} &= -\sigma \dot{\chi}_{(t-\delta t,\vec{x})} + D_i^- D_i^+ \chi_{(t,\vec{x})} - \lambda_2 |\phi_{(t,\vec{x})}|^2 (|\chi_{(t,\vec{x})}|^2 - \eta_2^2) \chi_{(t,\vec{x})}, \\ \Delta_t A_{i,(t+\delta t,\vec{x})} &= \dot{A}_{i,(t,\vec{x})}, \\ \Delta_t C_{i,(t+\delta t,\vec{x})} &= \dot{C}_{i,(t,\vec{x})}, \\ \Delta_t \phi_{(t+\delta t,\vec{x})} &= \dot{\phi}_{(t,\vec{x})}, \\ \Delta_t \chi_{(t+\delta t,\vec{x})} &= \dot{\chi}_{(t,\vec{x})}, \end{aligned} \quad (\text{B.3})$$

where $\Delta_t \phi_{(t)} = \delta t^{-1} [\phi_{(t)} - \phi_{(t-\delta t)}]$ etc.

In the temporal gauge, the fields also have to satisfy the Gauss laws,

$$\begin{aligned} \sum_i \Delta_i^- \dot{A}_{i,(t,\vec{x})} &= 2e_1 \text{Im} \phi_{(t,\vec{x})}^* \dot{\phi}_{(t,\vec{x})}, \\ \sum_i \Delta_i^- \dot{C}_{i,(t,\vec{x})} &= 2e_2 \text{Im} \chi_{(t,\vec{x})}^* \dot{\chi}_{(t,\vec{x})}, \end{aligned} \quad (\text{B.4})$$

which appear as additional constraints for the initial conditions.

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